PG – 150

III Semester M.Sc. Examination, December 2015
(CBCS)
MATHEMATICS
M303T : Fluid Mechanics

Time : 3 Hours Max. Marks : 70

*Instruction : Answer **any five full** questions.*

1. a) Show that :

   i) If $a_i b_j$ are components of a vector then a_{ij} are components of a tensor.

   ii) A tensor is orthogonal iff $\underset{\sim}{A}\vec{a} \cdot \underset{\sim}{A}\vec{b} = \vec{a} \cdot \vec{b}$ for all vectors $\vec{a}$ and $\vec{b}$. Also deduce that $\left|\underset{\sim}{A}\vec{a}\right| = |\vec{a}|$ for all vectors if $\underset{\sim}{A}$ is orthogonal. (4+4)

   b) State and prove divergence theorum for a tensor field $\underset{\sim}{A}$. 6

2. a) Define : Path lines, stream lines and Vortex lines.
   If the acceleration is the gradient of a scalar function, then show that the circulation round a material curve remains constant in time t. 8

   b) With usual notation, show that $\frac{D}{Dt}\int_V \phi\, dv = \int_V \frac{\partial \phi}{\partial t}\, dv + \int_S \phi\, \vec{q} \cdot \hat{n}\, dS$. 6

3. a) Derive the equation of continuity in the form : $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{q}) = 0$ and hence show that $\frac{D}{Dt}\int_V \rho\phi\, dv = \int_V \rho \frac{D\phi}{Dt}\, dv$, where the quantities have their usual meaning. 7

   b) Using an appropriate conservation law show that the stress tensor is symmetric. 7

4. a) With usual notation derive Navier Stokes equation. 8

b) For a certain flow of a non-viscous fluid of constant density under the Earth's gravitational field, the velocity distribution is given by $\vec{q} = -\nabla\phi$, where $\phi = x^3 - 3xy^2$. Find the pressure distribution. 6

5. a) State and prove Kelvin's minimum energy theorem. 8

b) Define any two non-dimensional numbers and discuss their physical significance. 6

6. a) Discuss the flow whose complex potential is given by $W = Uz + m\,\text{Ln}\,(z-a) - m\,\text{Ln}\,(z+a)$. 7

b) Find the image system of a doublet. 7

7. State and prove Blasius theorem and any one of its major applications. 14

8. Obtain exact solution of the Navier-Stokes equation for the following problems :

a) Generalized Plane-Couette flow and

b) Stokes' first problem. (7+7)